eigenvalues, each corresponding to a propagating impedance mode of the multiconductor line.

ACKNOWLEDGMENT

The idea of generalizing the properties of inhomogeneous lines to inhomogeneous multiconductor lines was originated by K. Mannersalo.

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Equivalent Circuits of Binomial Form Nonuniform Coupled Transmission Lines

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Abstract—**Equivalent circuits of nonuniform coupled transmission lines whose self and mutual characteristic admittance distributions obey binomial form are presented. Telegrapher's equations of these nonuniform coupled transmission lines can be solved exactly using Bessel functions of fractional order. By decomposing the chain matrix, it is shown that equivalent circuits of these nonuniform coupled transmission lines consist of cascade connections of lumped reactance elements, uncoupled uniform transmission lines and ideal transformers.**

I. INTRODUCTION

COUPLED TRANSMISSION lines are very important in microwave network theory. They are used extensively in all types of microwave components: filters, couplers, matching sections, and equalizers. Uniform coupled transmission lines have been described by many authors [1]–[15], and it is well known that equivalent representations of coupled transmission lines are very significant techniques in the analysis and synthesis. Nonuniform coupled transmission lines show good transmission responses and may also be important in microwave network theory. In general, the analysis of nonuniform coupled transmission lines becomes hard work because of difficulty of finding exact network functions. The analysis of particular nonuniform coupled transmission lines, for instance, exponential or hyperbolic tapered coupled transmission lines, have been reported [16], [17], but useful equivalent representations have not been obtained.

In this paper, we investigate equivalent circuits of nonuniform coupled transmission lines whose self and mutual characteristic admittance distributions obey binomial form. First, it is shown that telegrapher's equations of these nonuniform coupled transmission lines can be solved exactly using Bessel functions of fractional order. Then, by decomposing chain matrices of these circuits, we can show that equivalent circuits of these nonuniform coupled transmission lines are expressed as cascade connections of lumped reactance elements, uncoupled uniform transmission lines and ideal transformers. Two-port equivalent circuits of parabolic tapered coupled transmission lines with appropriate terminal conditions imposed are also presented by using equivalent representations shown in this paper.

II. EXACT SOLUTIONS OF TELEGRAPHER'S EQUATIONS

The $2n$th-order binomial form coupled transmission lines (BFCTL) are nonuniform coupled transmission lines whose self and mutual characteristic admittance distributions are given as the binomial form $(ax+b)^{\pm 2n}$, where x is the distance along the line, a and b are constants and n is an integer. The lossless $2n$th-order BFCTL above a ground

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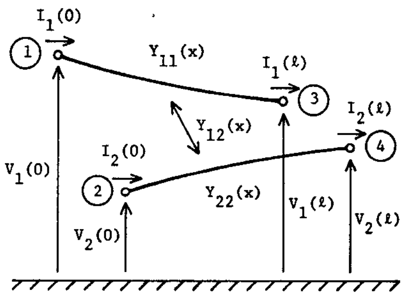


Fig. 1. Two-wire binomial form coupled transmission lines above a ground plane.

plane, shown in Fig. 1, may be described by the following equations:

$$\begin{bmatrix} -\frac{d}{dx}V_1(x) \\ -\frac{d}{dx}V_2(x) \end{bmatrix} = \frac{j\omega}{\left(1+\frac{x}{k}\right)^{2n}} \cdot \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} I_1(x) \\ I_2(x) \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} -\frac{d}{dx}I_1(x) \\ -\frac{d}{dx}I_2(x) \end{bmatrix} = j\omega\left(1+\frac{x}{k}\right)^{2n} \cdot \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} V_1(x) \\ V_2(x) \end{bmatrix} \quad (2)$$

where

$V_i(x)$ the voltage across ith transmission line at $x=x$;
$I_i(x)$ the current in ith transmission line at $x=x$;
L_{ii} self inductance of ith transmission line at $x=0$;
L_{ij} mutual inductance between ith and jth transmission lines at $x=0$;
C_{ii} self capacitance of ith transmission line at $x=0$;
C_{ij} mutual capacitance between ith and jth transmission lines at $x=0$ ($i, j=1,2$);
ω the angular frequency;
k constant.

A. The Balanced Mode (Odd Mode)

The voltage $V_b(x)$ and the current $I_b(x)$ for the balanced mode (odd mode) are expressed as follows:

$$\left.\begin{aligned} V_b(x) &= V_1(x) - V_2(x) \\ I_b(x) &= I_1(x) = -I_2(x) \end{aligned}\right\}. \quad (3)$$

We can obtain the following telegrapher's equations using (1), (2), and (3):

$$\left.\begin{aligned} &\frac{d^2}{dx^2}V_b(x) + \frac{2n}{k+x}\cdot\frac{d}{dx}V_b(x) + \beta_b^2\cdot V_b(x) = 0 \\ &\frac{d^2}{dx^2}I_b(x) - \frac{2n}{k+x}\cdot\frac{d}{dx}I_b(x) + \beta_b^2\cdot I_b(x) = 0 \end{aligned}\right\}. \quad (4)$$

β_b is the phase constant for the balanced mode and is given by

$$\beta_b = \omega\sqrt{L_b C_b} = \omega\sqrt{\mu\epsilon} \equiv \beta \quad (5)$$

where

$$\left.\begin{aligned} L_b &= L_{11} + L_{22} - 2L_{12} \\ C_b &= \frac{C_{11}C_{22} - C_{12}^2}{C_{11} + C_{22} + 2C_{12}} \end{aligned}\right\} \quad (6)$$

μ is permeability

and

ϵ is permittivity.

General solutions of (4) are obtained by

$$\left.\begin{aligned} V_b(y) &= K_1\cdot y^{-(2n-1)/2}\cdot J_{(2n-1)/2}(\beta y) + (-1)^n\cdot K_2\cdot y^{-(2n-1)/2}\cdot J_{-(2n-1)/2}(\beta y) \\ I_b(y) &= K_3\cdot y^{(2n+1)/2}\cdot J_{(2n+1)/2}(\beta y) - (-1)^n\cdot K_4\cdot y^{(2n+1)/2}\cdot J_{-(2n+1)/2}(\beta y) \end{aligned}\right\} \quad (7)$$

where

$$y = k + x \quad (8)$$

$J_{(2n+1)/2}(\beta y)$ = Bessel function of fractional order

and K_i ($i=1-4$) are constants. Therefore, a chain matrix for the balanced mode is given as follows:

$$\begin{bmatrix} V_b(0) \\ I_b(0) \end{bmatrix} = \begin{bmatrix} \mathbb{A}_b & \mathbb{B}_b \\ \mathbb{C}_b & \mathbb{D}_b \end{bmatrix} \begin{bmatrix} V_b(l) \\ I_b(l) \end{bmatrix} \quad (9)$$

$$\begin{aligned} \mathbb{A}_b = T\cdot M\{&J_{-(2n-1)/2}(\beta k)\cdot J_{(2n+1)/2}(\beta(k+l)) \\ &+ J_{(2n-1)/2}(\beta k)\cdot J_{-(2n+1)/2}(\beta(k+l))\} \end{aligned} \quad (10)$$

$$\begin{aligned} \mathbb{B}_b = -j\frac{1}{Y_b}\cdot\frac{T}{M}\{&J_{-(2n-1)/2}(\beta k)\cdot J_{(2n-1)/2}(\beta(k+l)) \\ &- J_{(2n-1)/2}(\beta k)\cdot J_{-(2n-1)/2}(\beta(k+l))\} \end{aligned} \quad (11)$$

$$\begin{aligned} \mathbb{C}_b = jY_b\cdot T\cdot M\{&J_{-(2n+1)/2}(\beta k)\cdot J_{(2n+1)/2}(\beta(k+l)) \\ &- J_{(2n+1)/2}(\beta k)\cdot J_{-(2n+1)/2}(\beta(k+l))\} \end{aligned} \quad (12)$$

$$\begin{aligned} \mathbb{D}_b = \frac{T}{M}\{&J_{-(2n+1)/2}(\beta k)\cdot J_{(2n-1)/2}(\beta(k+l)) \\ &+ J_{(2n+1)/2}(\beta k)\cdot J_{-(2n-1)/2}(\beta(k+l))\} \end{aligned} \quad (13)$$

where

$$\left.\begin{aligned} T&=(-1)^n\cdot\sqrt{\frac{\pi\beta k}{2}}\sqrt{\frac{\pi\beta(k+l)}{2}}\\ M&=\left(\frac{k+l}{k}\right)^n\end{aligned}\right\}\tag{14}$$

l is the line length

and Y_b is the characteristic admittance for the balanced mode at $x=0$.

$$Y_b=\sqrt{\frac{C_b}{L_b}}=\frac{\sqrt{\mu\epsilon}}{L_{11}+L_{22}-2L_{12}}=\frac{y_{11}y_{22}-y_{12}^2}{y_{11}+y_{22}+2y_{12}}\tag{15}$$

where

y_{ii} self characteristic admittance of ith transmission line at $x=0$; and

y_{ij} mutual characteristic admittance between ith and jth transmission lines at $x=0$ $(i, j=1,2)$.

B. The Unbalanced Mode (Even Mode)

The voltage $V_u(x)$ and the current $I_u(x)$ for the unbalanced mode (even mode) are expressed as follows:

$$\left.\begin{aligned} V_u(x)&=V_1(x)=V_2(x)\\ I_u(x)&=I_1(x)+I_2(x)\end{aligned}\right\}.\tag{16}$$

Substituting (16) in (1) and (2), we get

$$\left.\begin{aligned} -\frac{d}{dx}V_u(x)&=j\omega\frac{L_u}{\left(1+\frac{x}{k}\right)^{2n}}I_u(x)\\ -\frac{d}{dx}I_u(x)&=j\omega\cdot C_u\cdot\left(1+\frac{x}{k}\right)^{2n}V_u(x)\end{aligned}\right\}\tag{17}$$

where

$$\left.\begin{aligned} L_u&=\frac{L_{11}L_{22}-L_{12}^2}{L_{11}+L_{22}-2L_{12}}\\ C_u&=C_{11}+C_{22}+2C_{12}\end{aligned}\right\}.\tag{18}$$

The telegrapher's equation for the unbalanced mode is the same expression as (4) with the b subscripts changed to u. The phase constant β_u for the unbalanced mode is defined by

$$\beta_u=\omega\sqrt{L_uC_u}=\omega\sqrt{\mu\epsilon}\equiv\beta.\tag{19}$$

Accordingly, the chain matrix for the unbalanced mode is given as follows:

$$\begin{bmatrix} V_u(0)\\ I_u(0)\end{bmatrix}=\begin{bmatrix}\mathbb{A}_u & \mathbb{B}_u\\ \mathbb{C}_u & \mathbb{D}_u\end{bmatrix}\begin{bmatrix} V_u(l)\\ I_u(l)\end{bmatrix}\tag{20}$$

where $\mathbb{A}_u$, $\mathbb{B}_u$, $\mathbb{C}_u$, and $\mathbb{D}_u$ are identical to (10), (11), (12), and (13), respectively, with the b subscripts changed to u. The characteristic admittance Y_u for the unbalanced mode at $x=0$ is given by

$$Y_u=\sqrt{\frac{C_u}{L_u}}=\frac{1}{\sqrt{\mu\epsilon}}(C_{11}+C_{22}+2C_{12})=y_{11}+y_{22}+2y_{12}.\tag{21}$$

C. The Chain Matrix of the Four-Port Network

The voltage V_i and the current I_i $(i=1,2)$ of the four-port network shown in Fig. 1 are expressed as follows:

$$\begin{bmatrix} V_1\\ V_2\end{bmatrix}=\begin{bmatrix} 1 & \frac{\delta}{1+\delta}\\ 1 & -\frac{1}{1+\delta}\end{bmatrix}\begin{bmatrix} V_u\\ V_b\end{bmatrix}\tag{22}$$

$$\begin{bmatrix} I_1\\ I_2\end{bmatrix}=\begin{bmatrix}\frac{1}{1+\delta} & 1\\ \frac{\delta}{1+\delta} & -1\end{bmatrix}\begin{bmatrix} I_u\\ I_b\end{bmatrix}\tag{23}$$

where

$$\delta=-\frac{V_1}{V_2}=\frac{I_2}{I_1}.\tag{24}$$

By substituting (9) and (20) in (22) and (23), the chain matrix $[F]$ of the four-port network is obtained as follows:

$$\begin{bmatrix} V_1(0)\\ V_2(0)\\ I_1(0)\\ I_2(0)\end{bmatrix}=[F]\begin{bmatrix} V_1(l)\\ V_2(l)\\ I_1(l)\\ I_2(l)\end{bmatrix}\tag{25}$$

$$[F]=\begin{bmatrix}\mathbb{A}_{11} & 0 & \mathbb{B}_{11} & \mathbb{B}_{12}\\ 0 & \mathbb{A}_{11} & \mathbb{B}_{21} & \mathbb{B}_{22}\\ \mathbb{C}_{11} & \mathbb{C}_{12} & \mathbb{D}_{11} & 0\\ \mathbb{C}_{21} & \mathbb{C}_{22} & 0 & \mathbb{D}_{11}\end{bmatrix}\tag{26}$$

$$\begin{aligned}\mathbb{A}_{11}=T\cdot M\{&J_{-(2n-1)/2}(\beta k)\cdot J_{(2n+1)/2}(\beta(k+l))\\ &+J_{(2n-1)/2}(\beta k)\cdot J_{-(2n+1)/2}(\beta(k+l))\}\end{aligned}\tag{27}$$

$$\begin{aligned}\mathbb{B}_{rs}=-j\frac{y_{sr}}{y_{11}y_{22}-y_{12}^2}\frac{T}{M}\{&J_{-(2n-1)/2}(\beta k)\\ &\cdot J_{(2n-1)/2}(\beta(k+l))-J_{(2n-1)/2}(\beta k)\\ &\cdot J_{-(2n-1)/2}(\beta(k+l))\}\qquad (r,s=1,2)\end{aligned}\tag{28}$$

$$\begin{aligned}\mathbb{C}_{rs}=j(-1)^{r+s}\cdot y_{rs}\cdot T\cdot M\{&J_{-(2n+1)/2}(\beta k)\\ &\cdot J_{(2n+1)/2}(\beta(k+l))\\ &-J_{(2n+1)/2}(\beta k)\\ &\cdot J_{-(2n+1)/2}(\beta(k+l))\}\qquad (r,s=1,2)\end{aligned}\tag{29}$$

$$\begin{aligned}\mathbb{D}_{11}=\frac{T}{M}\{&J_{-(2n+1)/2}(\beta k)\cdot J_{(2n-1)/2}(\beta(k+l))\\ &+J_{(2n+1)/2}(\beta k)\cdot J_{-(2n-1)/2}(\beta(k+l))\}.\end{aligned}\tag{30}$$

Thomson's polynomial $H_n(z)$ is defined by

$$H_n(z)=U_n(z)+G_n(z)=z^n\cdot\left[u_n\left(\frac{1}{z}\right)+g_n\left(\frac{1}{z}\right)\right]\tag{31}$$

where

$$\tanh(z)=\frac{\epsilon^z-\epsilon^{-z}}{\epsilon^z+\epsilon^{-z}}\equiv\frac{U_n(z)}{G_n(z)}\tag{32}$$

and

$$\left.\begin{aligned} u_n\left(\frac{1}{z}\right) &= \frac{1}{z^n}\cdot U_n(z) \\ g_n\left(\frac{1}{z}\right) &= \frac{1}{z^n}\cdot G_n(z) \end{aligned}\right\}. \tag{33}$$

Bessel functions of fractional order are expressed using modified Thomson's polynomial as

$$\left.\begin{aligned} \sqrt{\frac{\pi\beta k}{2}}\cdot J_{(2n+1)/2}(\beta k) &= (j)^n\cdot g_n\left(\frac{1}{z}\right) \\ &\quad\cdot\sin(\beta k) + (j)^{n+1} \\ &\quad\cdot u_n\left(\frac{1}{z}\right)\cdot\cos(\beta k) \\ \sqrt{\frac{\pi\beta k}{2}}\, J_{-(2n+1)/2}(\beta k) &= (-j)^{n+1}\cdot u_n\left(\frac{1}{z}\right) \\ &\quad\cdot\sin(\beta k) + (-j)^n \\ &\quad\cdot g_n\left(\frac{1}{z}\right)\cdot\cos(\beta k) \end{aligned}\right\} \tag{34}$$

where

$$z = j\beta k. \tag{35}$$

By substituting (34) in (27)–(30), we can decompose the chain matrix of (26) as follows:

$$[F] = [F_1]\cdot[F_2]\cdot[F_3]\cdot[F_4] \tag{36}$$

where

$$[F_1] = \begin{bmatrix} g_{n-1}\left(\frac{1}{z}\right)\cdot[I] & u_{n-1}\left(\frac{1}{z}\right)\cdot[Y]^{-1} \\ g_n\left(\frac{1}{z}\right)\cdot[Y] & u_n\left(\frac{1}{z}\right)\cdot[I] \end{bmatrix} \quad (n=\text{odd}) \tag{37}$$

and

$$[F_1] = \begin{bmatrix} u_{n-1}\left(\frac{1}{z}\right)\cdot[I] & g_{n-1}\left(\frac{1}{z}\right)\cdot[Y]^{-1} \\ u_n\left(\frac{1}{z}\right)\cdot[Y] & g_n\left(\frac{1}{z}\right)\cdot[I] \end{bmatrix} \quad (n=\text{even}) \tag{38}$$

$[I]$ is the 2×2 identity matrix, and

$$[Y] = \begin{bmatrix} y_{11} & -y_{12} \\ -y_{21} & y_{22} \end{bmatrix} \tag{39}$$

$$[F_2] = \begin{bmatrix} \cos(\beta l)\cdot[I] & j\sin(\beta l)\cdot[Y]^{-1} \\ j\sin(\beta l)\cdot[Y] & \cos(\beta l)\cdot[I] \end{bmatrix} \tag{40}$$

$$[F_3] = \begin{bmatrix} u_n\left(\frac{1}{z'}\right)\cdot[I] & -u_{n-1}\left(\frac{1}{z'}\right)\cdot[Y]^{-1} \\ -g_n\left(\frac{1}{z'}\right)\cdot[Y] & g_{n-1}\left(\frac{1}{z'}\right)\cdot[I] \end{bmatrix} \quad (n=\text{odd}) \tag{41}$$

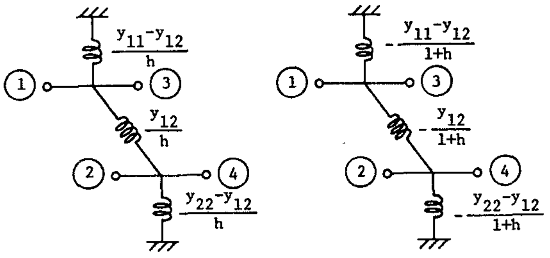


Fig. 2. Lumped inductance coupled circuits.

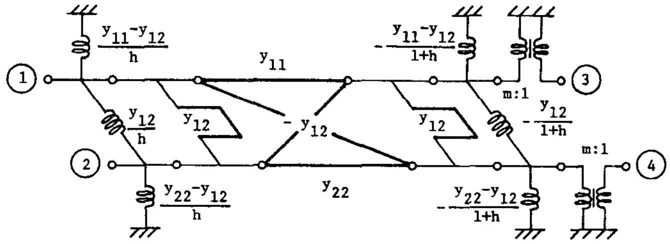


Fig. 3. The equivalent circuit of the second-order binomial form coupled transmission lines.

$$[F_3] = \begin{bmatrix} g_n\left(\frac{1}{z'}\right)\cdot[I] & -g_{n-1}\left(\frac{1}{z'}\right)\cdot[Y]^{-1} \\ -u_n\left(\frac{1}{z'}\right)\cdot[Y] & u_{n-1}\left(\frac{1}{z'}\right)\cdot[I] \end{bmatrix} \quad (n=\text{even}) \tag{42}$$

where

$$z' = j\beta(k+l) \tag{43}$$

and

$$[F_4] = \begin{bmatrix} M & 0 & 0 & 0 \\ 0 & M & 0 & 0 \\ 0 & 0 & M^{-1} & 0 \\ 0 & 0 & 0 & M^{-1} \end{bmatrix}. \tag{44}$$

III. Equivalent Circuits

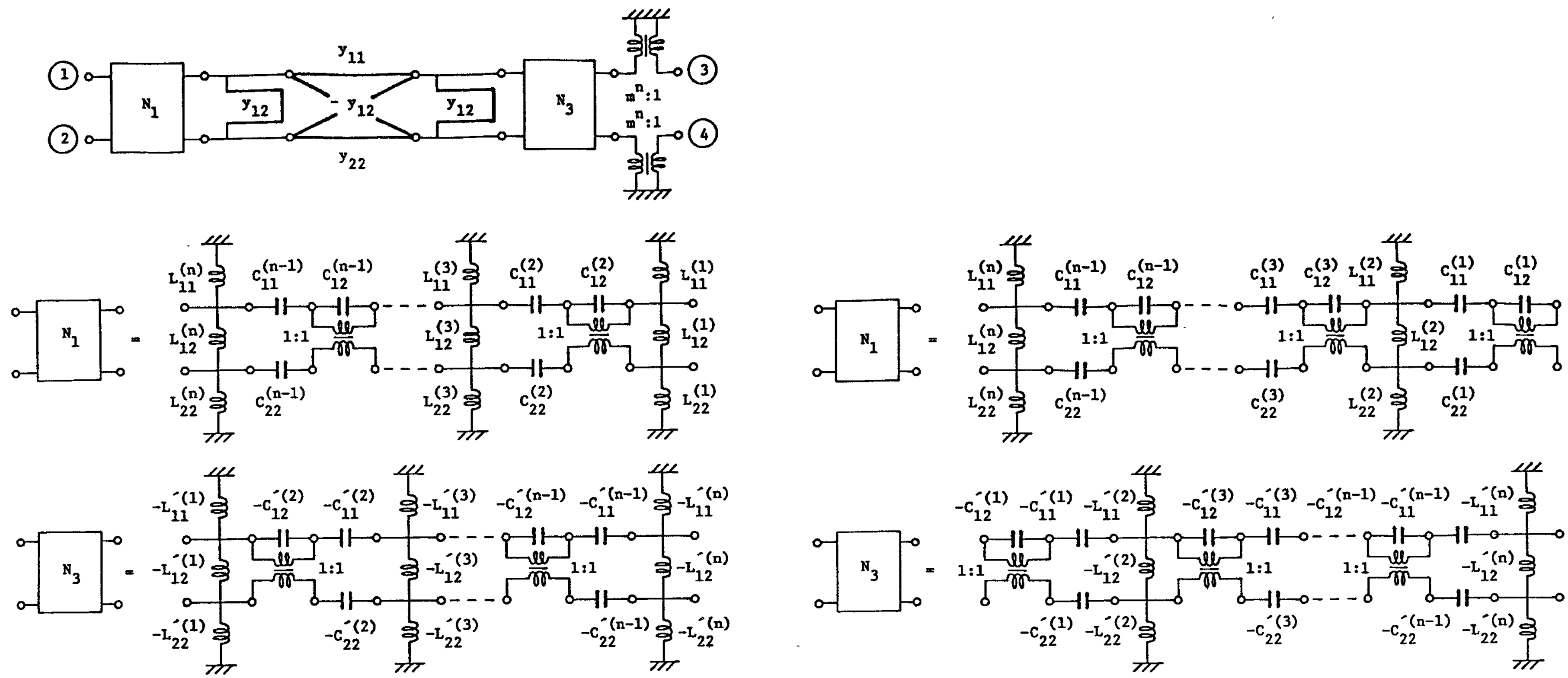
Chain matrix $[F_2]$ is the one for uniform coupled transmission lines, and the equivalent circuit of uniform coupled transmission lines is expressable by uncoupled transmission lines [10]. Chain matrix $[F_4]$ expresses an ideal transformer bank. If chain matrices $[F_1]$ and $[F_3]$ express appropriate circuits, we can obtain an equivalent circuit with cascade structure for BFCTL. We define a taper coefficient h and a transformation ratio m of BFCTL as follows:

$$h = k/l \tag{45}$$

$$m = (1+h)/h. \tag{46}$$

In the case of $n=1$, chain matrices (37) and (41) are expressed by

$$[F_1] = \begin{bmatrix} [I] & [0] \\ \frac{1}{s}\cdot\frac{1}{h}\cdot[Y] & [I] \end{bmatrix} \tag{47}$$



$$L_{11}^{(i)} = \frac{h}{(2i-1)(y_{11}-y_{12})}, \quad L_{12}^{(i)} = \frac{h}{(2i-1)\,y_{12}}, \quad L_{22}^{(i)} = \frac{h}{(2i-1)(y_{22}-y_{12})}$$

$$C_{11}^{(j)} = \frac{h}{(2j-1)(z_{11}-z_{12})}, \quad C_{12}^{(j)} = \frac{h}{(2j-1)\,z_{12}}, \quad C_{22}^{(j)} = \frac{h}{(2j-1)(z_{22}-z_{12})}$$

(i=odd , j=even)

$$L_{11}^{\prime(i)} = \frac{1+h}{(2i-1)(y_{11}-y_{12})}, \quad L_{12}^{\prime(i)} = \frac{1+h}{(2i-1)\,y_{12}}, \quad L_{22}^{\prime(i)} = \frac{1+h}{(2i-1)(y_{22}-y_{12})}$$

$$C_{11}^{\prime(j)} = \frac{1+h}{(2j-1)(z_{11}-z_{12})}, \quad C_{12}^{\prime(j)} = \frac{1+h}{(2j-1)\,z_{12}}, \quad C_{22}^{\prime(j)} = \frac{1+h}{(2j-1)(z_{22}-z_{12})}$$

(i=odd , j=even).

(a)

$$C_{11}^{(i)} = \frac{h}{(2i-1)(z_{11}-z_{12})}, \quad C_{12}^{(i)} = \frac{h}{(2i-1)\,z_{12}}, \quad C_{22}^{(i)} = \frac{h}{(2i-1)(z_{22}-z_{12})}$$

$$L_{11}^{(j)} = \frac{h}{(2j-1)(y_{11}-y_{12})}, \quad L_{12}^{(j)} = \frac{h}{(2j-1)\,y_{12}}, \quad L_{22}^{(j)} = \frac{h}{(2j-1)(y_{22}-y_{12})}$$

(i=odd , j=even)

$$C_{11}^{\prime(i)} = \frac{1+h}{(2i-1)(z_{11}-z_{12})}, \quad C_{12}^{\prime(i)} = \frac{1+h}{(2i-1)\,z_{12}}, \quad C_{22}^{\prime(i)} = \frac{1+h}{(2i-1)(z_{22}-z_{12})}$$

$$L_{11}^{\prime(j)} = \frac{1+h}{(2j-1)(y_{11}-y_{12})}, \quad L_{12}^{\prime(j)} = \frac{1+h}{(2j-1)\,y_{12}}, \quad L_{22}^{\prime(j)} = \frac{1+h}{(2j-1)(y_{22}-y_{12})}$$

(i=odd , j=even).

(b)

Fig. 4. Equivalent circuits of the $2n$th-order binomial form coupled transmission lines. (a) n=odd. (b) n=even.

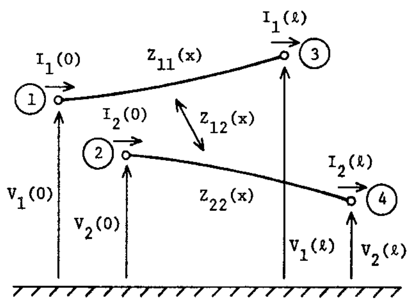
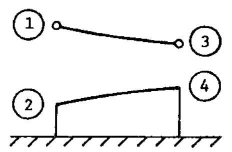
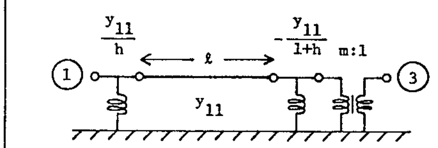
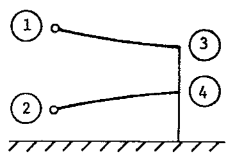
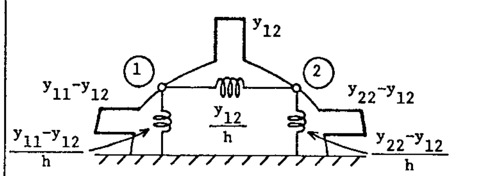
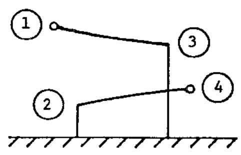
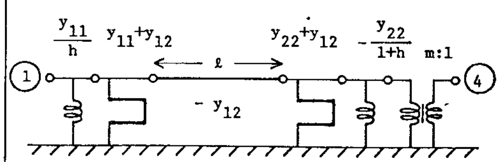


Fig. 5. Binomial form coupled transmission lines, the dual of the circuit shown in Fig. 1.

TABLE I
TWO-PORT EQUIVALENT CIRCUITS OF THE NETWORK SHOWN IN FIG. 1

	ORIGINAL CIRCUIT	EQUIVALENT CIRCUIT
1		
2		
3		

and

$$[F_3]=\begin{bmatrix} [I] & [0] \\ -\frac{1}{s}\cdot\frac{1}{1+h}\cdot[Y] & [I] \end{bmatrix} \tag{48}$$

where [0] is the 2×2 zero matrix, and

$$s=j\beta l. \tag{49}$$

The circuits realized from (47) and (48) are shown in Figs. 2(a) and (b), respectively. Therefore, the equivalent circuit of BFCTL in the case of $n=1$ (namely, parabolic tapered coupled transmission lines) becomes the mixed lumped and distributed circuit shown in Fig. 3. Fig. 3 represents a unit element of length l, and the short-circuited stub is also of length l.

In general, an equivalent circuit for BFCTL for any odd n becomes the circuit shown in Fig. 4(a), where the characteristic impedance z_{ij} is given by

$$z_{ij}=\frac{y_{ji}}{y_{11}y_{22}-y_{12}^2} \qquad (i,j=1,2). \tag{50}$$

An equivalent circuit for BFCTL for any even n is shown in Fig. 4(b).

The BFCTL shown in Fig. 5 is the dual of the circuit shown in Fig. 1. In the same manner as the derivations above, we can obtain equivalent circuits of these BFCTL as mixed lumped and distributed circuits. These are shown in Figs. 6(a) and (b), where element values of the circuit shown in Fig. 6(a) are given by those of the circuit shown in Fig. 4(b), and values of the circuit shown in Fig. 6(b) are given by those of the circuit shown in Fig. 4(a). The line length of the open-circuited stub equals l.

IV. EXAMPLES OF TWO-PORT NETWORK

Several two-port equivalent circuits of BFCTL can be derived, in the case of $n=1$, by using equivalent representations of Fig. 3 and Fig. 6(a). The first boundary condition is that two of the terminal voltages be zero. Two-port equivalent circuits from the network shown in Fig. 1 are introduced in Table I. The second boundary condition is that two of the terminal currents be zero. Two-port equivalent circuits of the network shown in Fig. 5 are introduced

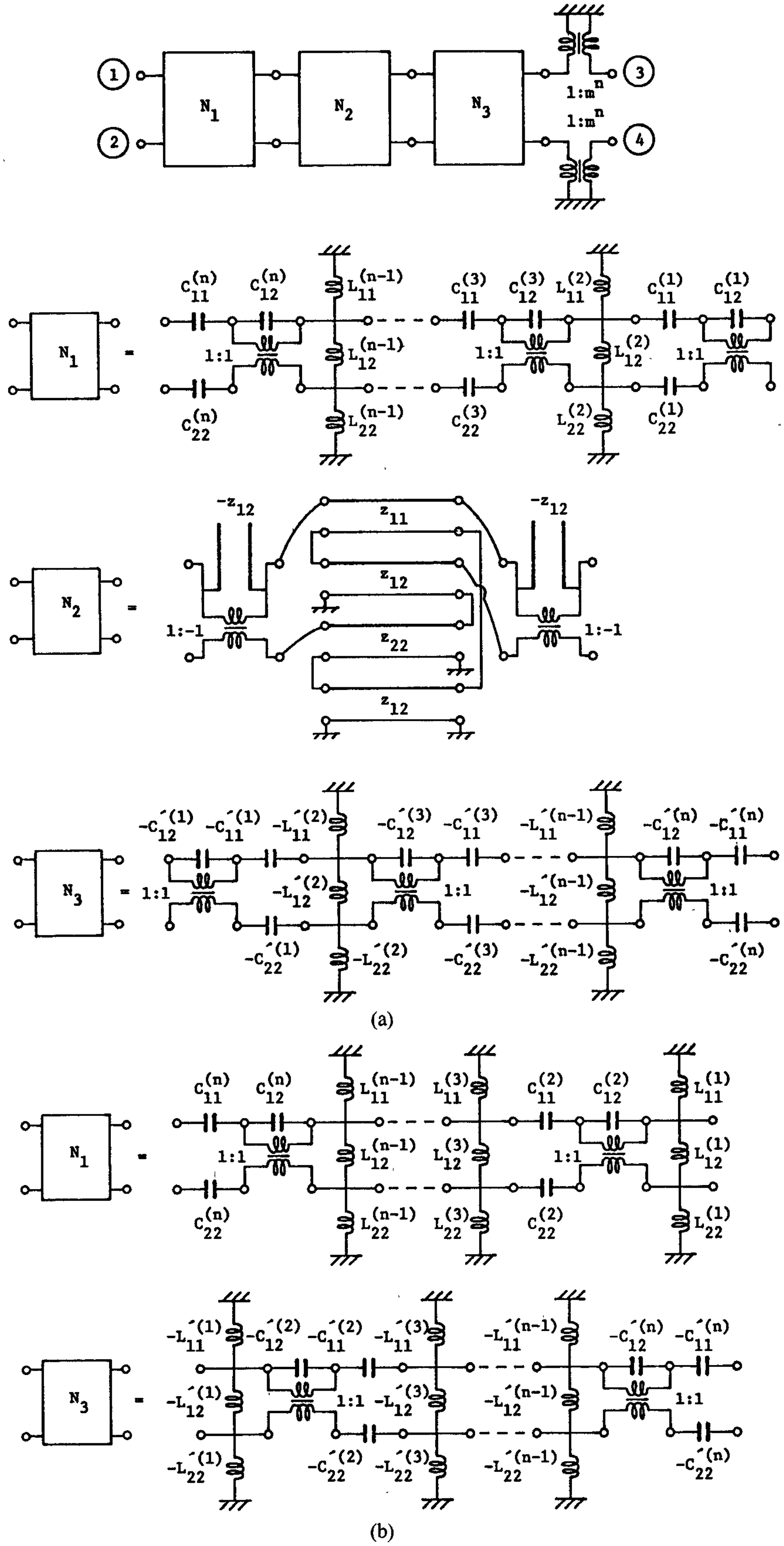
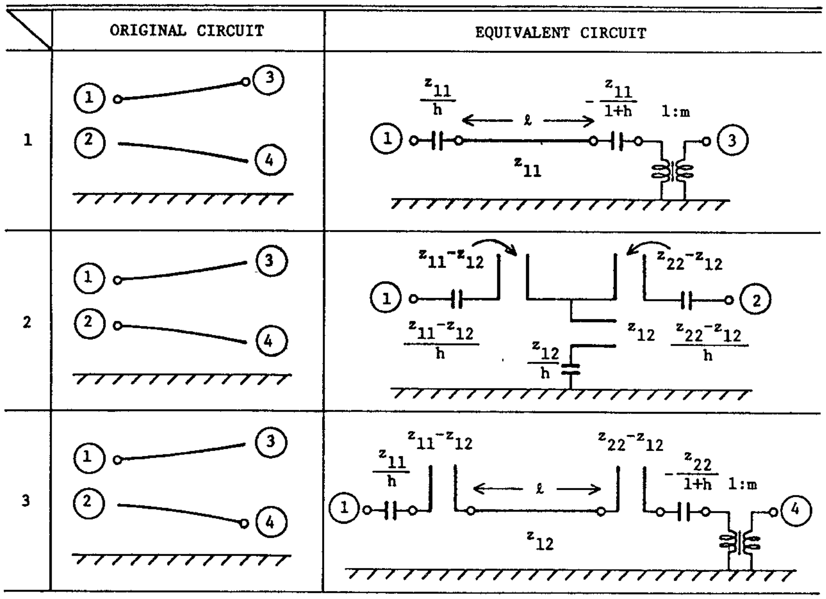


Fig. 6. Impedance type equivalent circuits of the $2n$th-order binomial form coupled transmission lines. (a) n=odd. (b) n=even.

TABLE II
TWO-PORT EQUIVALENT CIRCUITS OF THE NETWORK SHOWN IN FIG. 5



in Table II. Circuits shown in Table I-1 and Table II-1 are equivalent circuits of binomial form nonuniform transmission lines whose characteristic admittance distributions are

$$Y(x)=y_{11}\cdot\left(1+\frac{1}{h}\cdot\frac{x}{l}\right)^2 \tag{51}$$

$$Y(x)=\frac{1}{z_{11}\cdot\left(1+\frac{1}{h}\cdot\frac{x}{l}\right)^2} \tag{52}$$

respectively [18].

From Tables I and II, it can be seen that two-port BFCTL's are applicable to various microwave components. It is hoped that the equivalent circuits described here may lead to easier design methods for these microwave components.

V. CONCLUSION

Equivalent circuits of a class of nonuniform coupled transmission lines have been derived. Telegrapher's equations of the $2n$th-order BFCTL can be solved exactly using Bessel functions of fractional order. Then, by decomposing chain matrices of these networks, equivalent circuits of BFCTL are represented as mixed lumped and distributed circuits consisting of cascade connections of lumped reactance elements, uncoupled uniform transmission lines and ideal transformers. Therefore, BFCTL can be treated in the same manner as uniform coupled transmission lines, and potential applications using BFCTL may be expected. Finally, several two-port equivalent circuits of parabolic tapered coupled transmission lines with simple terminal conditions imposed are presented.

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